THE ENERGY BALANCE IN TURBULENT MOTION IN A PLANE CURVILINEAR CHANNEL

D. A. Nusupbekova and B. P. Ustimenko

We determine the component energy equations for mean and pulsating motion in a plane curvilinear channel. We discuss the characteristics of the turbulence in curvilinear channels by comparison with those of rectilinear flows.

Consider the turbulent circular flow of an incompressible fluid in a plane curvilinear channel. As shown in [1] at a sufficient distance from the entrance (for $\varphi > 120°$) a profile (Fig. 1) is established in the curvilinear channel, the nondimensional velocity of which $v_\varphi/v_{\varphi\max}$ has become constant and does not change with increasing distance along the channel. The diagram also shows the distribution of tangential friction stresses in a transverse section of the channel (u = v in the captions of Figs. 1, 3).

The following equations hold for such a flow:

$$v_x = v_x', \quad v_r = v_r'$$
$$v_\varphi = \langle v_\varphi \rangle + v_\varphi', \quad p = \langle p \rangle + p'$$
$$\langle v_x \rangle = \langle v_r \rangle = 0, \quad \langle v_\varphi \rangle = \langle v_\varphi \rangle (r)$$
$$p = \langle p \rangle (r, \varphi)$$

The derivatives with respect to x and φ of the averaged variables (with the exception of $dp/d\varphi \neq 0$) are zero. Here and hereafter $\langle v_x \rangle$, $\langle v_r \rangle$, $\langle v_\varphi \rangle$, $\langle p \rangle$ are (Reynolds) averages; v_x', v_r', v_φ', and p' are the pulsation values of the axial, radial, and tangential velocity components and the static pressure.

We write down the equations we need in what follows:

the energy balance of the mean motion

$$\left\{\frac{\partial}{\partial T}\frac{E}{v_{*1}^2}\right\}_1 + \left\{\frac{\langle v_\varphi \rangle}{v_{*1}}\frac{1}{R}\frac{\partial \Delta \langle p \rangle}{\partial \varphi}\right\}_2 - \left\{\frac{1}{N_{\mathrm{Re}}}\frac{\langle v_\varphi \rangle}{v_{*1}}\left[\frac{d}{dR}\left(\frac{d}{dR}\frac{\langle v_\varphi \rangle}{v_{*1}} + \frac{1}{R}\frac{\langle v_\varphi \rangle}{v_{*1}}\right)\right.\right.$$
$$\left.\left. + \frac{2}{R}\left(\frac{d}{dR}\frac{\langle v_\varphi \rangle}{v_{*1}} - \frac{1}{R}\frac{\langle v_\varphi \rangle}{v_{*1}}\right)\right]\right\}_3 + \left\{\frac{1}{R}\frac{d}{dR}\left(R\frac{\langle v_\varphi \rangle}{v_{*1}}\frac{\langle v_r' v_\varphi' \rangle}{v_{*1}^2}\right)\right\}_4 \tag{1}$$
$$- \left\{\frac{\langle v_r' v_\varphi' \rangle}{v_{*1}^2}\left(\frac{d}{dR}\frac{\langle v_\varphi \rangle}{v_{*1}} - \frac{\langle v_\varphi \rangle}{v_{*1}}\frac{1}{R}\right)\right\}_5 = 0$$

the pulsating energy in the direction x, r, φ

$$\frac{1}{2}\frac{\partial}{\partial T}\frac{\langle v_x'^2 \rangle}{v_{*1}^2} - \frac{p'}{\rho v_{*1}^2}\frac{\partial}{\partial X}\frac{v_x'}{v_{*1}} + \frac{1}{N_{\mathrm{Re}}}\left[\left\langle\left(\frac{\partial}{\partial X}\frac{v_x}{v_{*1}}\right)^2\right\rangle - \left\langle\left(\frac{\partial}{\partial R}\frac{v_x'}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{1}{R}\frac{\partial}{\partial \varphi}\frac{v_x'}{v_{*1}}\right)^2\right\rangle\right] = 0 \tag{2}$$

$$\frac{1}{2}\frac{\partial}{\partial T}\frac{v_r'^2}{v_{*1}^2} - 2\frac{\langle v_r' v_\varphi' \rangle}{v_{*1}^2}\frac{1}{R}\frac{\langle v_\varphi \rangle}{v_{*1}} - \left\langle\frac{p}{\rho v_{*1}^2}\frac{\partial}{\partial R}\frac{v_r'}{v_{*1}}\right\rangle \tag{3}$$
$$+ \frac{1}{N_{\mathrm{Re}}}\left[\left\langle\left(\frac{\partial}{\partial X}\frac{v_r'}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{\partial}{\partial R}\frac{v_r'}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{1}{R}\frac{\partial}{\partial \varphi}\frac{v_r'}{v_{*1}}\right)^2\right\rangle\right] = 0$$

$$\left\langle\frac{1}{2}\frac{\partial}{\partial T}\frac{v_\varphi'^2}{v_{*1}^2}\right\rangle + \left\langle\frac{v_r' v_\varphi'}{v_{*1}^2}\right\rangle\frac{1}{R}\frac{\partial}{\partial R}\left(\langle R \rangle\frac{\langle v_\varphi \rangle}{v_{*1}}\right) - \left\langle\frac{p'}{\rho v_{*1}^2}\frac{1}{R}\frac{\partial}{\partial \varphi}\frac{v_\varphi'}{v_{*1}}\right\rangle \tag{4}$$
$$+ \frac{1}{N_{\mathrm{Re}}}\left[\left\langle\left(\frac{\partial}{\partial X}\frac{v_\varphi'}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{\partial}{\partial R}\frac{v_\varphi'}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{1}{R}\frac{\partial}{\partial \varphi}\frac{v_\varphi'}{v_{*1}}\right)^2\right\rangle\right] = 0$$

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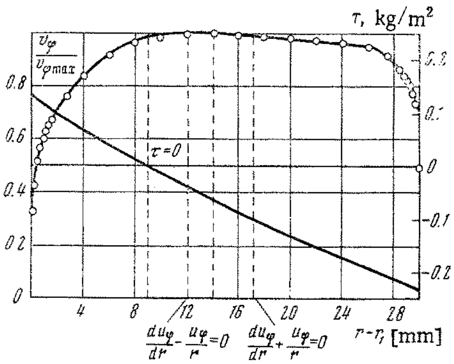

Fig. 1

the complete kinetic energy balance of the turbulence

$$\left\{\frac{\partial}{\partial T}\frac{E}{v_{*1}^2}\right\}_1 + \left\{\left\langle\frac{v_r' v_\varphi'}{v_{*1}}\right\rangle\left(\frac{d}{dR}\frac{\langle v_\varphi\rangle}{v_{*1}} - \frac{1}{R}\frac{\langle v_\varphi\rangle}{v_{*1}}\right)\right\}_2 + \left\{\frac{1}{N_{\mathrm{Re}}}\left[\left\langle\left(\frac{\partial}{\partial X}\frac{v_\varphi'}{v_{*1}}\right)^2\right\rangle\right.\right.$$
$$+\left\langle\left(\frac{\partial}{\partial R}\frac{v_x'}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{1}{R}\frac{\partial}{\partial\varphi}\frac{v_x'}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{\partial}{\partial X}\frac{v_r'}{v_{*1}^2}\right)^2\right\rangle + \left\langle\left(\frac{\partial}{\partial R}\frac{v_r'}{v_{*1}}\right)^2\right\rangle$$
$$\left.\left.+\left\langle\left(\frac{1}{R}\frac{\partial}{\partial\varphi}\frac{v_r'}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{\partial}{\partial X}\frac{v_\varphi'}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{\partial}{\partial R}\frac{v_\varphi}{v_{*1}}\right)^2\right\rangle + \left\langle\left(\frac{1}{R}\frac{\partial}{\partial\varphi}\frac{v_\varphi'}{v_{*1}}\right)^2\right\rangle\right]\right\}_3 = 0$$

$$X = \frac{x}{r_2 - r_1},\ R = \frac{r}{r_2 - r_1},\ T = \frac{tv_{*1}}{r_2 - r_1} \tag{5}$$

Here

$$\overline{E} = 0.5\,(\langle v_x^2\rangle + \langle v_r^2\rangle + \langle v_\varphi^2\rangle),\qquad E = 0.5\,(\langle v_x'^2\rangle + \langle v_r'^2\rangle + \langle v_\varphi'^2\rangle)$$

are the kinetic energies of the mean and pulsating motions, X, R, T are nondimensional coordinates, $v_{*1} = \sqrt{\tau_1/\rho}$ is the dynamical velocity, τ_1 is the frictional stress at the convex channel wall, N_{Re} is the Reynolds criterion.

It is easy to see that Eqs. (2)-(5) for pulsating motion completely coincide with the analogous equations for circular motion between co-axial rotating cylinders [2] while the energy balance equation for the mean motion (1) differs only by the presence of an additional term connected with the pressure forces (1).

From Eqs. (2)-(5) we have omitted, because of the smallness of the terms, the convective transfer of the energy of the turbulence of the mean motion and also viscous and turbulent diffusion. The last two terms, as shown in [3], only play a marked role in the laminar and transitional boundary layer.

The first term $\{\cdot\}_1$ in Eq. (1) has the physical meaning of the local change in the kinetic energy of the mean motion, $\{\cdot\}_2$ is the action of the pressure forces, the terms $\{\cdot\}_3$ and $\{\cdot\}_4$ are associated with the action of the viscous and turbulent stresses, while $\{\cdot\}_5$ describes the conversion of energy between the mean and pulsating flows.

The second term $\{\cdot\}_2$ in Eq. (5) has a similar meaning but it occurs there with opposite sign to the term in equation (1) [cf. the terms $\{\cdot\}_5$ in Eq. (1)]. Here the first term indicates the local change in the energy of the turbulence, while the last term $\{\cdot\}_3$ indicates the viscous dissipation of the pulsation energy.

All the terms in the energy balance Eqs. (1)-(5) can be computed using the experimental determination of the velocity distribution, the pressure and the tangential friction stresses [1] at sections of the plane curvilinear channel shown in Fig. 1.

The results of these computations for established flow in regions at the convex and concave walls of the channel and in the center are shown in Figs. 2, 3, and 4.

The pressure gradient along the axis of the curvilinear channel produces a confined circular flow of the fluid. As a result of the action of pressure forces, there is an increase in the kinetic energy of the mean motion in the whole flow domain (curves 2 in Figs. 2, 3, and 4). In the central zone the mean flow loses its kinetic energy under the influence of turbulent shear stresses (curve 4 in Fig. 4). A part of the kinetic energy of the mean motion is transformed here into the energy of the turbulence (curve 5).

The generation of turbulence increases sharply in the direction of the walls of the curvilinear channel. The action of the viscous shear stresses in this region of the flow is negligibly small (curve 3).

In the boundary layers of the flow the components of energy balance associated with the action of the viscous shear stresses and with the generation of pulsating motion increases sharply (3 and 5 in Figs. 2 and 4). Energy losses due to these effects are compensated by energy increases as a result of the action of the turbulent shear stresses 4. This term passes through zero near the wall and becomes negative, which corresponds to an increase in the kinetic energy of the mean motion.

Thus, in the boundary layer of the curvilinear channel there is a flow of kinetic energy of the mean motion which is transformed there into the energy of turbulence. The energy of the turbulence generated by the mean motion is dissipated and eventually is turned into heat. The generation of pulsating energy and its dissipation are virtually balanced over a large part of the section of the channel.

Discussion of the pulsating energy balance equations in various directions makes it possible to determine certain characteristics of the turbulence in curvilinear channels by comparison with rectilinear flows.

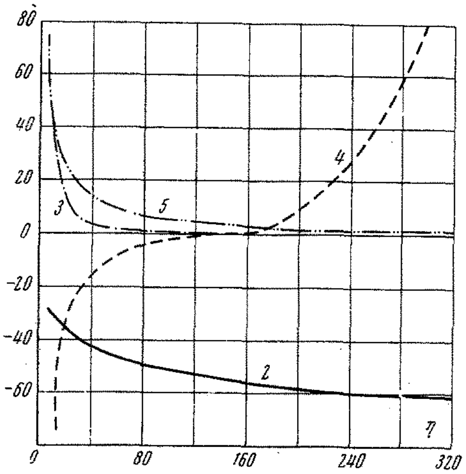

Fig. 2

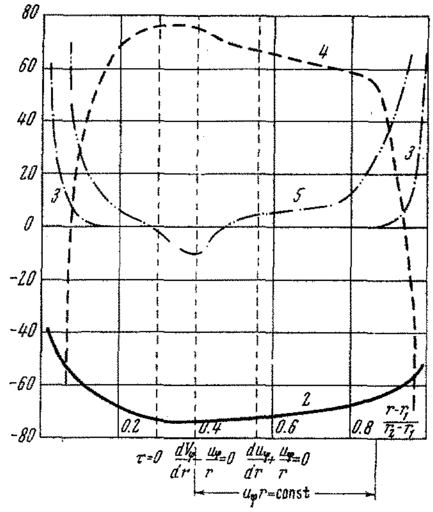

Fig. 3

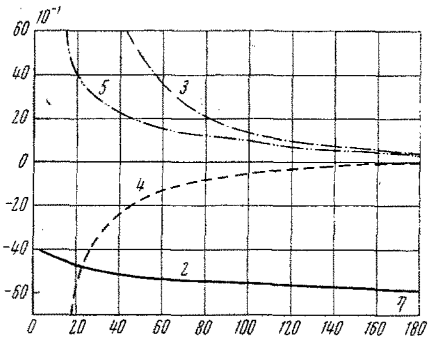

Fig. 4

First, as distinct from the flow in a rectilinear channel, turbulence is generated not only in the direction of the fundamental motion $\langle v_\varphi'^2\rangle$, but also in the radial direction $\langle v_r'^2\rangle$.

The curvilinearity of the flow affects in various ways the generation of pulsating energy in the radial direction in regions near the convex and concave walls [2]. Near the convex wall (the region of stable stratification of the flow) the correlation $\langle v_r' v_\varphi'\rangle$ is negative, while the second term in Eq. (3) is positive. This implies that the turbulence is suppressed and $\langle v_r'^2\rangle$ decreases. The converse pattern is observed near the concave wall (the region of unstable stratification of the flow), where the correlation $\langle v_r' v_\varphi'\rangle$ is positive. Here the mean motion generates turbulence and $\langle v_r'^2\rangle$ increases.

A reduction in the intensity of the turbulent motion at the convex wall and an increase at the concave wall are fully in agreement with the intensification of the processes of turbulent transfer observed in experiments [1, 4] and so of the hydrodynamical friction drag and heat transfer at the concave wall and correspondingly with their reduction at the convex wall.

As distinct from flows in rectilinear channels, where the term for the generation of turbulence in the fundamental direction of the motion [the second term in Eq. (4)] is always negative, which corresponds to an increase in the intensity of the turbulence, in a curvilinear channel there is a region in which suppression of the turbulence $\langle v_r'^2\rangle$ is observed by the averaged motion.

Indeed, since the points at which $\langle v_r' v_\varphi'\rangle = 0$, do not coincide, we have

$$\frac{d\langle v_\varphi\rangle}{dr} + \frac{\langle v_\varphi\rangle}{r} = 0$$

Between them there is a region (Fig. 1) in which the second term in Eq. (4) is positive, which indicates suppression of the intensity of the pulsations $\langle v_r'^2\rangle$ as a result of the action of the turbulent shear stresses.

A similar pattern is also observed for the complete energy of turbulence. In the region of the flow between the points (Fig. 1) where

$$\langle v_r' v_\varphi'\rangle = 0, \quad \frac{d\langle v_\varphi\rangle}{dr} - \frac{\langle v_\varphi\rangle}{r} = 0$$

term 2 of Eq. (5) is positive and term 5 of Eq. (1) is negative. From this there follows what is at first glance an unexpected conclusion that in this region the turbulent energy decreases and, correspondingly, the energy of the mean motion increases, i.e., there is a transformation of energy from the disordered pulsating motion to the ordered mean motion. The presence in the flow of such a region is clearly shown in Fig. 3 [cf. the region between the points $(r - r_1)(r_2 - r_1) = (0.3$ and $0.5]$.

With a certain degree of similarity these phenomena occur in accelerating rectilinear flows [3]. In a flow accelerating in the direction of motion there is a tendency for a reduction in the intensity of the turbulence and in decelerating flows a tendency for an increase.

A similar pattern, as indicated in [5], is observed also in atmospheric turbulence on the scale of the general circulation of the atmosphere, where observation cannot be explained without assumptions about the transformation of the energy of irregular perturbations into the energy of averaged flow in certain regions.

It should also be noted that the same characteristics can also apparently be observed in other turbulent flows in which the positions of zero correlations $\langle v'_i v'_j \rangle$ and the derivative of the velocity $(d\langle v_j \rangle / dx_j)$ do not coincide, for example, in turbulent boundary layers [6].

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